EXERCISE – V

JEE PROBLEMS

1. (a) If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$

then f(100) is equal to **[JEE 99, 2+10]**

(A) 0 (B) 1 (C) 100 (D) – 100

(b) Let a, b, c, d be real numbers in G.P. If u, v, w satisfy the system of equations u + 2v + 3w = 6 ; 4u + 5v + 6w = 12 then show that the roots of the equation $6u + 9v = 4\left(\frac{1}{u}+\frac{1}{v}+\frac{1}{w}\right)x^2 + [(b-c)^2 + (c-a)^2 + (d-b)^2]\,x + u + v + w = 0$ and $20x^2 + 10(a-d)^2 x - 9 = 0$ are reciprocals of each other.

2. If the system of equations x – ky – z = 0, kx – y – z = 0, x + y – z = 0 has a non -zero solution then the possible values of k are **[JEE 2000 (Scr.)]**

(A) –1, 2 (B) 1, 2 (C) 0, 1 (D) –1, 1

3. Prove that for all values of θ **[JEE 2000(Mains),3]**

$$\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin(\theta+2\pi/3) & \cos(\theta+2\pi/3) & \sin(2\theta+4\pi/3) \\ \sin(\theta-2\pi/3) & \cos(\theta-2\pi/3) & \sin(2\theta-4\pi/3) \end{vmatrix} = 0$$

4. Find the real values of r for which the following system of linear equations has a non-trivial solutions. Also find the non-trivial solutions

$$2rx - 2y + 3z = 0$$
$$x + ry + 2z = 0$$
$$2x + rz = 0$$

[REE 2000 (Mains), 3]

5. Solve for x the equation $\begin{vmatrix} a^2 & a & 1 \\ \sin(n+1)x & \sin nx & \sin(n-1)x \\ \cos(n+1)x & \cos nx & \cos(n-1)x \end{vmatrix} = 0$

[REE 2001 (Mains), 3]

6. Test the consistency and solve them when consistent, the following system of equations for all values of λ

$$x + y + z = 1$$
$$x + 3y - 2z = \lambda$$
$$3x + (\lambda + 2)y - 3z = 2\lambda + 1$$

[REE 2001 (Mains), 5]

7. Let a, b, c be real number with $a^2 + b^2 + c^2 = 1$. Show that the equation

$\begin{vmatrix} ax-by-c & bx+ay & cx+a \\ bx+ay & -ax+by-c & cy+b \\ cx+a & cy+b & -ax-by+c \end{vmatrix} = 0$ represents a straight line. **[JEE 2001 (Mains), 6]**

8. The number of values of k for which the system of equations (k + 1) x + 8y = 4k, kx + (k + 3)y = 3k – 1 has infinitely many solutions is **[JEE 2002 (Scr.), 3]**

(A) 0 (B) 1 (C) 2 (D) infinite

9. If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where a, b, c are real positive numbers, abc = 1 and $A^TA = I$, then find the value of $a^3 + b^3 + c^3$. **[JEE 2003 (Mains), 2]**

10. The value of λ for which the system of equations 2x – y – z = 12, x – 2y + z = – 4, x + y + λz = 4 has no solution is **[JEE 2004 (Scr.)]**

(A) 3 (B) –3 (C) 2 (D) –2

11. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$ then the value of α is

(A) ± 3 (B) ± 2 (C) ± 5 (D) 0 **[JEE 2004 (Scr.)]**

12. If M is a 3 × 3 matrix, where det (M) = 1 and $M^TM = I$ (where 'I' is an identity matrix) then prove that det (M – I) = 0. **[JEE 2004, 2]**

13. $A=\begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}$, $B=\begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}$, $U=\begin{bmatrix} f \\ g \\ h \end{bmatrix}$ $V=\begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$, $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

If AX = U has infinitely many solution then prove that BX = V, can not have a unique solution. If further afd $\neq$ 0 then prove that BX = V has no solution. **[JEE 2004, 4]**

14. $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$, $I=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ & $A^{-1} = \left[\frac{1}{6}(A^2 + cA + dI)\right]$,

then the value of c and d are **[JEE 2005 (Scr.)]**

(A) (–6, –11) (B) (6, 11) (C) (–6, 11) (D) (6, –11)

15. If $P = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$ and $x = P^TQ^{2005}P$ then x is equal to **[JEE 2005 (Scr.)]**

(A) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 4+2005\sqrt{3} & 6015 \\ 2005 & 4-2005\sqrt{3} \end{bmatrix}$

(C) $\frac{1}{4}\begin{bmatrix} 2+\sqrt{3} & 1 \\ -1 & 2-\sqrt{3} \end{bmatrix}$ (D) $\frac{1}{4}\begin{bmatrix} 2005 & 2-\sqrt{3} \\ 2+\sqrt{3} & 2005 \end{bmatrix}$

16. Comprehension : Read the passage given below and answer the equations that follows.

Let A = $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, U_1, U_2 and U_3 are columns matrices

satisfying $AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ and U is

3 × 3 matrix whose columns are U_1, U_2, U_3 then answer the following questions. **[JEE 2006, 5 + 5 + 5]**

(a) The value of |U| is

(A) 3 (B) –3 (C) 3/2 (D) 2

(b) The sum of the elements of the matrix U^{-1} is

(A) –1 (B) 0 (C) 1 (D) 3

(c) The value of [3 2 0] U $\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ is

(A) 5 (B) 5/2 (C) 4 (D) 3/2

17. (a) Consider three points P = $(-\sin(\beta - \alpha), -\cos\beta)$, Q = $(\cos(\beta - \alpha), \sin\beta)$ and R = $(\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$, where $0 < \alpha, \beta, \theta < \pi/4$ **[JEE 2008, 3 + 3]**

(A) P lies on the line segment RQ

(B) Q lies on the line segment PR

(C) R lies on the line segment QP

(D) P, Q, R are non collinear

(b) Consider the system of equations $x - 2y + 3z = -1$

$$-x + y - 2z = k$$

$$x - 3y + 4z = 1.$$

Statement-I : The system of equation has no solution for $k \neq 3$.

because

Statement-II : The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$

(A) Statement-I is true, statement-II is true; statement-II is correct explanation for statement-I

(B) Statement-I is true, statement-II is true; statement-II is **NOT** correct explanation for statement-I

(C) Statement-I is true, Statement-II is False

(D) Statement-I is False, Statement-II is True

18. Match the following **[JEE 2008, 6]**

Column-I	Column-II
(A) The minimum value of $\frac{x^2+2x+4}{x+2}$ is	(P) 0
(B) Let A and B be 3 × 3 matrices of real numbers, where A is symmetric, B is skew-symmetric, and (A + B) (A – B) = (A – B) (A + B). If $(AB)^t = (-1)^k AB$, where $(AB)^t$ is the transpose of the matrix AB, then the possible values of k are	(Q) 1
(C) Let $a = \log_3 \log_3 2$. An integer k satisfying $1 < 2^{(-k+3^{-a})} < 2$, must be	(R) 2
(D) If $\sin\theta = \cos\phi$, then the possible values of $\frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right)$ are	(S) 3

19. Comprehension : Read the passage given below and answer the equations that follows.

Let A be the set of all 3 × 3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0. **[JEE 2009]**

(a) The number of matrices in A is

(A) 12 (B) 6 (C) 9 (D) 3

(b) The number of matrices A in A for which the system of linear equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has a unique solution, is

(A) less than 4 (B) at least 4 but less than 7

(C) at least 7 but less than 10 (D) at least 10

(c) The number of matrices A in A for which the system of linear equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is inconsistent, is

(A) 0 (B) more than 2 (C) 2 (D) 1

20. The number of 3 x 3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions is **[JEE 2010]**

(A) 0 (B) 2^9 - 1 (C) 168 (D) 2

21. Comprehension : Read the passage given below and answer the equations that follows.

Let p be an odd prime number and T_P be the following set of 2 x 2 matrices : **[JEE 2010]**

$$T_P = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix};\ a, b, c \in \{0, 1, 2, ..., p-1\} \right\}$$

(a) The number of A in T_P such that A is either symmetric or skew-symmetric or both, and det(A) divisible by p is
(A) $(p - 1)^2$ (B) $2(p - 1)$ (C) $(p - 1)^2 - 1$ (D) $2p - 1$

(b) The number of A in T_P such that the trace of A is not divisible by p but det (A) is divisible by p is
[**Note :** The trace of a matrix is the sum of its diagonal entries]
(A) $(p - 1)(p^2 - p + 1)$ (B) $p^3 - (p - 1)^2$
(C) $(p - 1)^2$ (D) $(p - 1)(p^2 - 2)$

(c) The number of A in T_P such that det(A) is not divisible by p is
(A) $2p^2$ (B) $p^3 - 5p$ (C) $p^3 - 3p$ (D) $p^3 - p^2$

22. Let k be a positive real number and let

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & 1 \end{bmatrix} \ \& \ B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$$

If det(adj A) + det(adj B) = 10^6, then {k} is equal to
{**Note :** adj M denotes the adjoint of a square matrix M and [k] denotes the largest integer less than or equal to k} **[JEE 2010]**

23. Let M and N be two 3 × 3 non-singular skew-symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2 N^2 (M^T N)^{-1} (MN^{-1})^T$ is equal to **[JEE 2011]**
(A) M^2 (B) $-N^2$ (C) $-M^2$ (D) MN

24. Let M be a 3 × 3 matrix satisfying

$$M\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix},\ M\begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix},\ \text{and } M\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\12\end{bmatrix}$$

Then the sum of the diagonal entries of M is **[JEE 2011]**

25. Let P = $[a_{ij}]$ be a 3 × 3 matrix and let Q = $[b_{ij}]$, where $b_{ij} = 2^{i+j} a_{ij}$ for $1 \le i, j \le 3$. If the determinant of P is 2, then the determinant of the matrix Q is **[JEE 2012]**
(A) 2^{10} (B) 2^{11} (C) 2^{12} (D) 2^{13}

26. If P is a 3 × 3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3 × 3 identity matrix, then there exists a column matrix $X = \begin{bmatrix}x\\y\\z\end{bmatrix} \ne \begin{bmatrix}0\\0\\0\end{bmatrix}$ such that **[JEE 2012]**

(A) $PX = \begin{bmatrix}0\\0\\0\end{bmatrix}$ (B) PX=X (C) PX=2X (D) PX = − X

27. If the adjoint of a 3 × 3 matrix P is $\begin{bmatrix}1 & 4 & 4\\2 & 1 & 7\\1 & 1 & 3\end{bmatrix}$, then the possible value(s) of the determinant of P is (are)
(A) −2 (B) −1 (C) 1 (D) 2 **[JEE 2012]**
Sol.